

# Sub-pixel Profiling

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**Abstract**—A procedure is described that enables a high resolution profile to be obtained across an edge or line. The method works by first performing a sub-pixel estimation of each point on the edge or line assuming a parabolic sub-pixel profile. A second stage fits a high order model along the length of the line, allowing any bias in the original sub-pixel fit to be estimated by applying a low order model to the residuals. Successive rows within the image are then fused to increase the sample rate and provide a sub-pixel profile of the line. The procedure is illustrated by obtaining the profile of a noisy edge.

**Keywords**—sub-pixel, super-resolution, gauging, line detecting, intensity profile

## I. INTRODUCTION

Many applications require, or can benefit from, locating features to sub-pixel accuracy. Examples are:

- Gauging: locating edges of objects to measure size for inspection.
- Camera calibration: locating feature points (lines, spots, edges) (eg [1-3]).
- Registration: determining offsets between objects for template matching, or calculating disparity in stereo imaging (eg [4]).
- Compression: determining offset for motion compensation.

Locating any feature to sub-pixel accuracy requires first of all that the feature can be reliably detected. This implies that the feature is sufficiently far (usually at least 2 pixels) from other similar features so that it can be resolved unambiguously. A second requirement is that the feature can be modelled with only a few parameters. By fitting the available data (pixel values) to the model, the location of the feature may be determined with accuracy significantly better than 1 pixel.

Correlation with a template results in a set of local maxima, the locations of correspond to locations of the template within the image. Alternatively, if minimum absolute difference is used, a set of local minima results [5].

Lines less than 2 pixels thick may be detected directly by looking perpendicular to the line. The line position corresponds to either the minima or maxima depending on whether it is a dark line against a light background, or vice versa. Edges may

be converted into lines by an appropriate edge detection filter [6].

When locating lines and edges to sub-pixel accuracy, the usual approach is to estimate each point on the line, and then fit a higher order model to these points. This is the general approach used within this paper to perform sub-pixel profiling. The fitting of the higher model to the detected edge or line points effectively forces consistency between those points, with the averaging process during the fit smoothing out noise resulting from the pixels occurring on a discrete grid. This two stage process is illustrated in Fig. 1.

The problem with this approach is that the detection of the feature points can introduce significant systematic error. While the second stage fitting process will tend to average these errors out, the assumption made by most fitting algorithms is that any noise is random. However the residuals from fitting are far from random, as shown in Fig. 2, and exhibit significant spatial correlation.

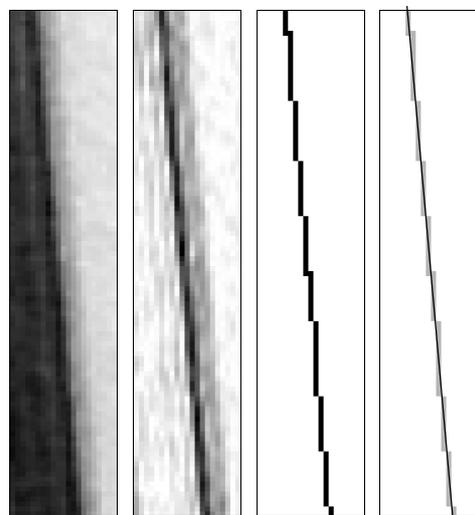


Figure 1. Two stage detection process for locating lines (or edges) to sub-pixel accuracy. If locating edges, a linear edge detection filter is applied to convert the edge into a line. Stage one locates feature points on the line to the nearest pixel, and stage two fits a higher order model to the detected points.

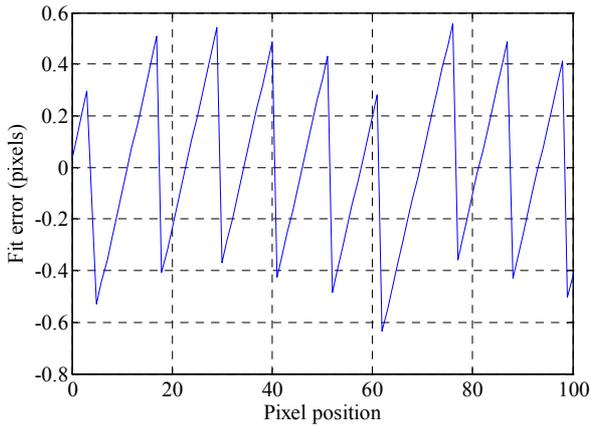


Figure 2. Residuals in fitting a line to the pixels detected in Fig. 1. A least squares fit assumes that the individual errors are random, whereas there is a strong correlation between adjacent errors. RMS residual = 0.3 pixel.

## II. MODELS FOR SUB-PIXEL ESTIMATION

This situation may be improved by locating each point on the line to sub-pixel accuracy, rather than to the nearest pixel. This will improve the accuracy of the first stage fitting process.

Since lines and edges are local features, it is best to use only local data, such as the maximum pixel and the one on either side. Therefore, given 3 (usually noisy) pixel values, the problem then becomes one of estimating the true position of the line or edge at that point on the edge. In detecting edges, the approach use here will be to apply an edge detection filter (such as a Sobel filter) to convert the edge to a line, and locate the position of the line to sub-pixel accuracy.

The notation used throughout this paper is illustrated in Fig. 3. A pixel's position is taken at the centre of that pixel. Pixels are numbered relative to the local extreme, which has a value of  $p_0$ . The pixel at 1 has value  $p_+$ , and that at  $-1$  has value  $p_-$ . The true extremum is located at a distance  $x$  from the extreme pixel value, where  $-0.5 < x < 0.5$ .

Using just these three pixel values, it is necessary to use a model of the profile across the line to give the value of  $x$ . Several different models have been proposed for this application [6].

The simplest model, for lack of any additional information is to fit a parabola to the 3 points, and use the position of the local maximum of the parabola as the estimate. This gives:

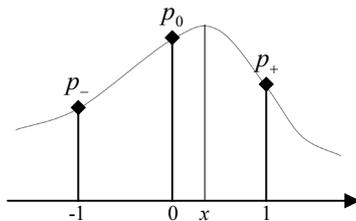


Figure 3. A profile across a line, showing the maximum pixel and the adjacent pixels. The problem is to estimate  $x$  given the 3 pixel values.

$$x = \frac{p_+ - p_-}{4p_0 - 2(p_+ + p_-)} \quad (1)$$

If, instead, the shape of the profile is considered to be triangular, the location of the peak is given by:

$$x = \frac{p_+ - p_-}{2(p_0 - \min(p_-, p_+))} \quad (2)$$

If the background can be assumed to be zero, the centre of gravity of the three sample pixels can give an estimate of the location:

$$x = \frac{p_+ - p_-}{p_0 + p_+ + p_-} \quad (3)$$

In most cases the background will no be zero. If the minimum of the flank pixels is assumed to be the background level, then the adjusted centre of gravity becomes

$$x = \frac{p_+ - p_-}{p_0 + p_+ + p_- - 3\min(p_+, p_-)} \quad (4)$$

This the same result, (4), is obtained if a rectangular section of exactly 1 pixel wide is assumed.

For detecting step edges with a Sobel filter or similar balanced 3x3 window filter, the local maximum will be at

$$x = \frac{p_+ - p_-}{2p_0} \quad (5)$$

Note that each of these models makes different assumptions about the sub-pixel profile, and will provide different estimates of the position of the local extremum. Which model is going to give the most accurate model for an arbitrary line with an unknown profile? This of course cannot be answered because an accurate model requires an accurate estimate of the sub-pixel profile. Using the wrong model will result in bias, introducing systematic errors that the second stage fit may not completely eliminate.

A second problem is that even if we know the sub-pixel profile, determining the equation of the peak of the profile is not easy. This is because the actual profile is not likely to fit a simple equation. With noise present, the problem is only exacerbated.

This results in a chicken and egg situation – we need an accurate estimate of the profile to be able to estimate the offset of each pixel accurately, and we need to know the offset of each pixel to be able to construct an accurate sub-pixel profile.

### III. GENERIC FIT MODEL

Since the fitting process in stage 2 will smooth or average much of the bias introduced from the first stage, a bootstrap approach may be used. We use one of the models in the previous section to estimate the sub-pixel offset, and then fit the higher order model across all of the pixels. This effectively refines the estimate, and provides an estimate of the bias introduced by the particular method.

Of the five methods described by (1) to (5) we want a method that is insensitive to both scaling and offset. Scaling invariance is important because as the light intensity changes, all of the pixel values will scale by a constant amount. All 5 methods above are scaling invariant. Offset invariance allows lines to be detected against a non-zero background. This would also make the system insensitive to the particular black level of the sensor. The methods described by (3) and (5) can be eliminated on this basis. The remaining equations all possess the property that they span the full range of  $x = -0.5$  when  $p_0 = p_-$  to  $x = 0.5$  when  $p_0 = p_+$ .

It is also convenient to be able to detect light lines on a dark background (for example after an edge detection filter), or dark lines on a white background (in the case of ink on a light surface). Equations (2) and (4) require a light line on a dark background. To reverse the intensities, the min in these equations must be replaced by a max. Since (1) works for both light on dark and dark on light, it is the simplest equation on which to base a generic model.

As (1) is fitting a parabola to the local extremum of the profile, any deviation from parabolic, or any asymmetry in the profile will result in systematic errors in the estimate. When the second stage fit is made to the line using the estimates generated by (1), the residuals are shown in Fig. 4.

The effect of using the sub-pixel fit of each line point is to significantly reduce the residual (from 0.3 to 0.11 pixel). The bias in using (1) may be estimated by comparing the residuals from stage 2 with the sub-pixel estimates of stage 1. This is shown in Fig. 5.

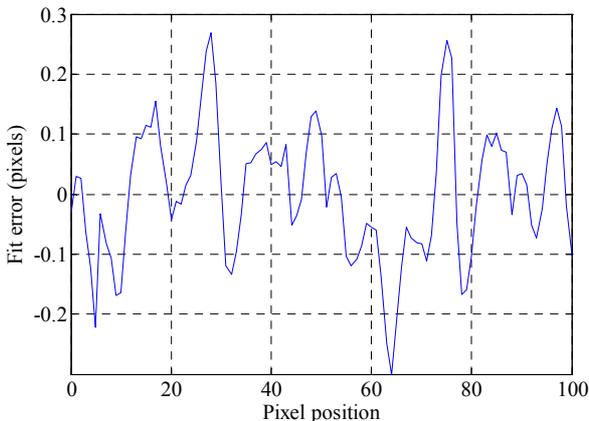


Figure 4. Residuals in fitting a line to the sub-pixel estimates of the line positions using (1). RMS residual 0.11 pixel.

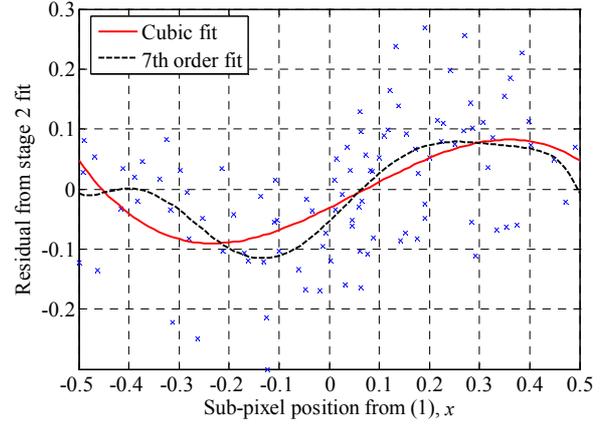


Figure 5. Residual as a function of sub-pixel fit from stage 1. The consistent pattern shows the bias in using (1) to estimate the sub-pixel positions. The cubic fitted to the bias reduces the residual to 0.094 pixel, while the 7<sup>th</sup> order fit reduced the residual to 0.090 pixel.

The residuals in Fig. 5 are not uniformly distributed, and result from the use of a parabolic approximation to the sub-pixel intensity profile. By fitting a curve to the residuals, it is possible to include much of the bias within the model.

$$Residual = f(x) \quad (6)$$

A constrained least squares fit may be used to model the bias. Since the  $x$  position from stage 1 will always be between -0.5 and 0.5, it is only necessary to model within this range. It is necessary to constrain the ends of this range for the following reason. Consider the situation when  $p_0 = p_+$ . The stage 1 fit using (1) gives  $x = 0.5$ . However, since these two pixel values are the same,  $p_+$  could just as easily have been selected as the centre pixel. In this case we would have  $p_0 = p_-$  which gives  $x = -0.5$ . The actual offset to the true line position does not change depending on which of the two equivalent pixel values we select as the maximum. The 1 pixel difference in position is exactly compensated for by (1) giving  $x = 0.5$  for one and -0.5 for the other situation. Therefore in fitting to the bias the constraint

$$f(0.5) = f(-0.5) \quad (7)$$

must be met. It is convenient to use a low order polynomial to model the bias in the absence of any additional information. The order of the polynomial should be chosen such that it reasonably models the bias without over-modelling and being overly influenced by the noise. This is particularly important if there are only a few data points. Consider the polynomial

$$f(x) = A_0 + A_1x + A_2x^2 + \dots = \sum_{n=0}^N A_n x^n \quad (8)$$

In applying the constraint of (7) the even order terms cancel out leaving

$$A_1 + \frac{A_3}{2^2} + \frac{A_5}{2^4} + \dots = 0 \quad (9)$$

The constraint therefore reduces the number of degrees of freedom of the fit by 1. Substituting (9) into (8) for a 3<sup>rd</sup> order (cubic) fit therefore gives

$$f(x) = A_0 + A_1(x - 4x^3) + A_2x^2 \quad (10)$$

Similarly a 5<sup>th</sup> order fit gives

$$f(x) = A_0 + A_1(x - 2^4x^5) + A_2x^2 + A_3(x^3 - 2^2x^5) + A_4x^4 \quad (11)$$

The least squares cubic fit is shown in Fig. 5. With this data, going to a higher order fit does not reduce the residual significantly, and introduces additional ripples. It is hard to say whether the additional ripples from higher order fits are an improvement of the fit, or are indicative of starting to overfit the data.

#### IV. SUB-PIXEL PROFILE

The previous section solved the problem of estimating the sub-pixel offset to an arbitrary profile. The next step is to use this information to obtain the actual profile.

For each point along the length of the line, the stage 2 fit effectively defines the sub-pixel position of the line at that point. The position of the line is defined as the origin of the profile, and the offset of each pixel from this centre is determined. In the example here, since the line is approximately vertical, the horizontal offset of each pixel from the line is determined. The ensemble of all of the samples near the line provides a large number of samples, all at potentially different offsets from the line. This increase in sample density allows a significant increase in the resolution across the line. The samples of the profile from the example in Fig. 1 are shown in Fig. 6.

With real data there is often significant noise within the image. In this example, the noise is very evident in Fig. 6. A number of techniques may be used to reduce or suppress the noise to obtain a representation of the underlying profile. Here we have used a smoothing spline.

In this example, the data was obtained from an edge by applying a linear edge detection filter. Applying the same ensembling process to the original edge data gives the sub-pixel profile across the edge. This is shown in Fig. 7, again with spline smoothing.

To validate the bias approximation method developed in the previous section, the smoothed profile curve was used derive the bias. Let the actual profile be  $p(x)$ . Then the sub-pixel approximation of the peak location given by (1) should relate directly to the offset used to calculate it:

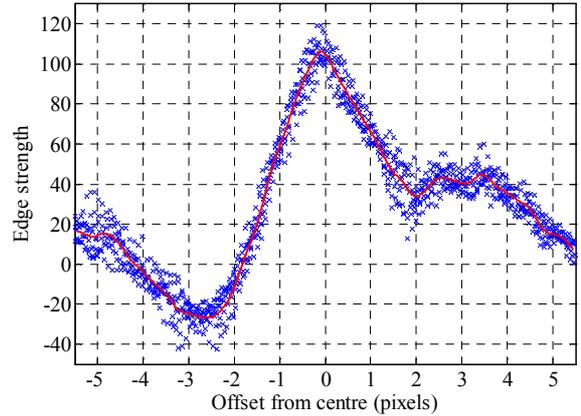


Figure 6. The intensity profile about the line in Fig. 1. The solid line is provided by applying a smoothing spline to the data.

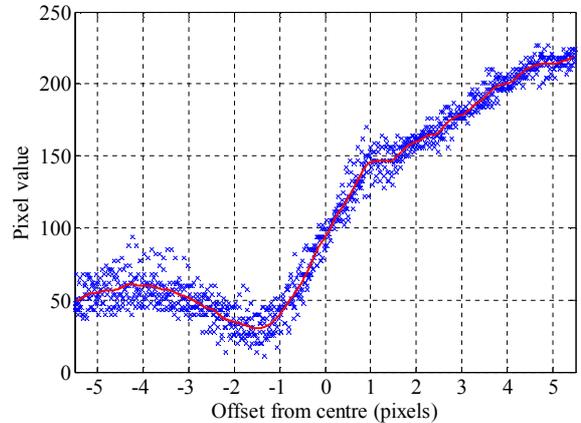


Figure 7. The intensity profile about the edge in Fig. 1. The solid line is the data smoothed by a smoothing spline.

$$-\hat{x} = \frac{p(x+1) - p(x-1)}{4p(x) - 2(p(x+1) + p(x-1))} \quad (12)$$

The bias in the approximation is therefore given by the deviation of the estimate from the actual value:

$$Bias = \hat{x} - x \quad (13)$$

Fig. 8 plots the bias against the sub-pixel estimate ( $-\hat{x}$ ) and compares this with the bias estimation derived by fitting to the residuals. Both the cubic and 7<sup>th</sup> order fits give reasonable approximations of the bias estimated by (13) given the level of noise in the image and in the residuals (in Fig. 5). One of the difficulties of a low order fit is that it cannot accurately represent the actual bias profile. Increasing the order allows a wider variation in the shape that can be represented, but this can be at the expense of overfitting. In that case, not all of the variation represented within the model may actually be present within the data.

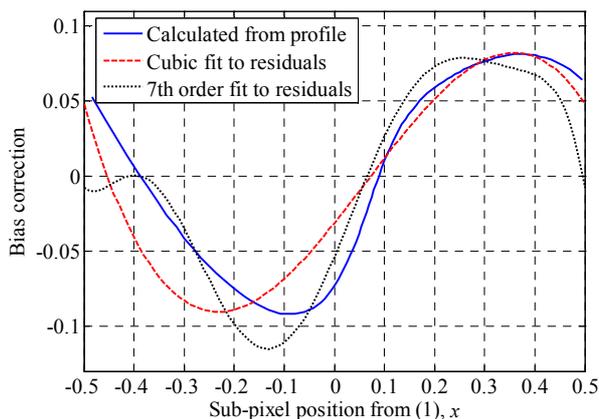


Figure 8. Validating the bias correction estimate. The solid line is derived directly from the smoothed profile in Fig. 6 using (12) and (13), while the other lines are estimates produced by fitting to the residuals.

## V. DISCUSSION AND SUMMARY

In this paper, a bootstrapping approach to estimating the sub-pixel profile is described. The line or edge being profiled must occupy several rows within the image, and the line or edge must be oriented at a slight angle to the pixel sample grid. The profile is constructed by combining together the samples over the several rows within the image. If an edge is being profiled, it must be converted into a line by applying a gradient filter. A sub-pixel fit is then used across each row of the image to estimate the position of the line to sub-pixel accuracy. A simple parabolic model is used initially. In the second stage, a smooth curve is fitted vertically through the sub-pixel line points. The residuals in this fit allow the bias resulting from using a parabolic model across the line profile. Fitting a low order polynomial to the residuals allows a generic correction term to be applied. This, combined with the parabolic model may be used to refine the smooth curve through all the points. The line or edge profile may then be constructed by offsetting each line from the image so that the second stage smooth fit becomes a vertical line.

The procedure described here is specific to vertical lines, although could easily be adapted to horizontal or diagonal lines.

The edges or lines do not necessarily need to be straight, although it is necessary to have a parametric model of the corresponding curve. Knowledge of the shape of the line or edge allows an appropriate model to be selected. Even if it is known that the line should be straight, any lens distortion in image will introduce curvature to the line. Therefore it is recommended that the second stage fit use at least a parabola or higher order curve. A parabola provides a reasonable approximation to the lens distortion if the distortion is mild [2] or the length of the line segment is sufficiently short.

The bias estimation stage can correct for an asymmetric sub-pixel profile. For example, the central peak in Fig. 6 is

mildly asymmetric. Any asymmetry in the actual data will be reflected in asymmetry in the correction term. This asymmetry is readily apparent in Fig. 8. When there is asymmetry, the peak of the profile may not correspond exactly to an offset of 0. However, since the sub-pixel profile is also constructed, the true location of the peak may be estimated and corrected. Shifting the profile is equivalent to adding a constant to the bias correction term.

In the example shown here, a parabolic model was used for the first stage fit. The models of (2) and (4) could equally be used with similar results. These different models would result in different bias functions. However, the first fit is used as a crude approximation, and the corresponding bias correction term generated from the residuals. In the absence of any detailed information about the actual profile, any of the models can provide a useful first estimate.

When estimating the bias correction, any noise in the data necessitates some form of smoothing function. In this paper, low order polynomial was used. In selecting the order of the polynomial, it is important to be able to model the key characteristics or shape of the bias without fitting unduly to the noise present. The actual polynomial order used would depend on the amount of noise present and also on the number of data points being fitted to. Other parametric models such as splines may be used, as long as the end constraint of (7) is satisfied.

If there is significant noise present in the image, the generated sub-pixel profile may be quite noisy. In this instance some form of noise smoothing may be useful. It should be kept in mind that any smoothing is also likely to blur the underlying profile.

With the profiles obtained using this approach, it must be remembered that the image is subject to blurring both by the lens and also by area sampling within the sensor. Both of these will introduce blur into the sub-pixel profile. If the actual edge or line profile is desired then it will be necessary to remove this blur by using some form of inverse filter.

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