Using Spectral Warping for Instrumentation and Measurements in Mixed-Signal Testing

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Abstract

Spectral Warping (SW) is the digital signal processing (DSP) procedure of transforming an original digital sequence to a new one having special spectral properties: equally spaced samples of its DFT (Discrete Fourier Transform) are identical to the unequally-spaced frequency samples of DFT of the original sequence. The use of SW can open up new opportunities in test signal generation as well as in test response analysis for mixed-signal circuits.

1. Introduction

One of the main problems in testing of analog and mixed-signal circuits is to find a minimum set of the internal test points of the Device-Under-Test and test frequencies that maximizes the testability [1]. This problem is usually solved when the circuit is being designed and synthesized. The other important problem is how to generate and apply such signals to the specified points of the *Device-Under-Test (DUT)*. It becomes even more complicated in the *Built-In Self-Test (BIST)* environment where available test resources are essentially limited.

This paper proposes to use special DSP procedure - Spectral Warping for *Mixed-Signal Test* generation and analysis, and discusses some specific features of the solution.

2. Spectral Warping

Spectral Warping is a special digital signal processing technique that was initially proposed by Oppenheim et. al. [2] and further elaborated by Oppenheim and Johnson [3] for wide range of applications (voice spectra correction, non-uniform frequency analysis using discrete orthogonal transforms, communications, etc.). In short the technique consists of transforming the original sequence to a new one having the property that equally spaced frequency samples of its *Discrete Fourier Transform (DFT)* are identical to unequally-spaced frequency samples of the DFT of the original sequence. This opens up the possibility of decreasing the data array size, and thereby increasing en efficiency of the discrete signal processing, as measured, for example, by speed of its execution.

The spectral warping network can be characterized as an all-pass digital filter cascade shown in Figure 1. The system function of such a multi-section network is

$$H_N(z)=\frac{(1-a^2)z^{-1}}{(1-az^{-1})}\left(\frac{z^{-1}-a}{1-az^{-1}}\right)^{N-1},N\geq 1$$

$$H_M(z)=\frac{1}{1-az^{-1}},N=0 \qquad (1)$$

where N is the number of a section in the network.

For real values of the parameter a (warping coefficient) the network produces signal frequency shift, which is described by the following expression

$$\varpi'=\frac{1}{T}\tan^{-1}\left[\frac{(1-a^2)\sin\varpi T}{(1+a^2)\cos\varpi T-2a}\right] \qquad (2)$$

where ω - input signal angular frequency; ω' - angular frequency of the output signal; T - sampling interval.

Figure 2 shows the frequency warping function for several values of the warp parameter a. It can bee seen that the frequency mapping between an original (network input) sequences and the output one is inherently nonlinear. Thus computing spectral values at frequency samples uniformly spaced along ω' (by using, for example, *Fast Fourier Transform (FFT)* algorithm) will correspond to the spectral analysis of the original

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sequence with non-uniformly spaced frequencies along ω. If parameter a is chosen to be real and positive between 0 and 1, the warping would lead to the higher spectral resolution at low frequencies. In contrast, a negative value of a between 0 and −1 would lead to the opposite effect, i.e., resolution at higher frequencies would be higher than that at the lower frequency band.

Amplitude- and phase-frequency characteristics of the spectral warping network are very important in order to apply the warping correctly. They were examined in [4] by analyzing the system function (1) of the network. The amplitude-frequency $H_M(\omega)$ and phase-frequency $\Phi_M(\omega)$ characteristics of the network consisting of M sections are given by:

$$|H_M(\varpi)| = \frac{1-a^2}{1-2a\cos\varpi T + a^2} \quad (3)$$

$$\Phi_M(\varpi) = -(M-1)\tan^{-1}\left[\frac{(1-a^2)\sin\varpi T}{(1+a^2)\cos\varpi T - 2a}\right] \quad (4)$$

It can be seen from the expression (3) and (4) that the effect of the frequency characteristics can be substantial. For example, the amplitude shows a ripple with period equal to the sampling frequency: the ratio of the amplitude at *dc* and the half-sampling frequency is $[(1+a)/(1-a)]^2$, the variation of which is shown in Fig. 3.

The spectral warping network and its frequency characteristics were experimentally evaluated by using computer simulation for the case of 1024 samples. The program flow chart is shown in Fig. 4. The following set of the recurrent expressions were used to describe the network operation [3]:

$$y_0(n) = a[y_0(n-1) - 0] + x(-n)$$
$$y_1(n) = a[y_1(n-1) - 0] + y_0(n-1) \quad (5)$$
$$y_k(n) = a[y_k(n-1) - y_k(n)], k = 2,3,....$$

where $\{y_k(n)\}$ – output (warped) sequence, $\{x(n)\}$ – input (original) sequence, a – coefficient of warping. Two arrays Y and YN were used in the program to store final and intermediate results in the SW network, while additional array was used to store the results of final spectral analysis by means of FFT (in fact, *FFT-Walsh*) algorithm. The experimental results fully confirmed the theoretical findings. For example, for a=0.9 warping of the input sequence having 30.07745 Hz dominant frequency and amplitude of 200 relative units led to the shift of the frequency peak to 117.3020 Hz while its amplitude was around 394 units, i.e., almost 2 times higher. This implies that the frequency characteristics should be certainly kept in mind when implementing the spectral warping network for different applications including that in testing.

The easiest and simplest way to eliminate the amplitude distortions cased by the spectral warping is to set coefficient a=0 in the first two sections of the network - it has been shown in [4] that the other sections are of the all-pass type, i.e., their amplitude characteristics are frequency-independent. However such an exclusion of the first two sections from the SW procedure could lead to the additional frequency distortions in the final spectra. We plan to analyzed the matter in our future research, and will report the results as soon as we get them. Much more reliable (though also more complicated) approach is to use programmable linear filter to pre-emphasize the frequency characteristics of the signal before warping. One of possible implementations of such a solution is presented below in Section 4.

3. Warping Network Implementation

One of the most important advantages of the spectral warping is that it can be implemented in a straightforward and inexpensive manner using just digital components. One of the possible forms of the implementation was shown in [3]. Based on the expressions (5) the network structure was redrawn to a new presented in Figure 5 indicating a realization with multipliers, adders, and delays (memory elements). It can be seen from the expressions that the arithmetical operations involved into the calculations are identical for iterations in both k and n. In particular, an arithmetic function of a type $A(a,b,c,d)=a(b-c)+d$ is required to realize computation in accordance with expressions (5). Such an arithmetic function can be implemented as a simple and inexpensive *arithmetic-logic unit (ALU)* with inputs a, b, c, d, and output A. Input a corresponds to the warping coefficient a. For the computation of $y_0(n)$ the inputs b, c, and d are $y_0(n-1)$, 0 and $x(-n)$ respectively, so that

$$y_0(n) = A[a, y_0(n-1), 0, x(-n)]$$
$$y_1(n) = A[a, y_1(n-1), 0, y_0(n-1)]$$
$$y_k(n) = A[a, y_k(n-1), y_{k-1}(n)], k = 2,3,... \quad (6)$$

The network of Figure 5, provides iteration in both k and n. The iteration in k corresponds to computing $y_k(n)$ along the network for fixed n. The iteration in n is related to computing the outputs for each new input sample. The storing of the delayed states of the network (presented by z^{-1} components) can naturally be realized with the use of a shift register. Block-diagram of the network hardware implementation using the shift register is shown in Figure 6 [3]. The state of the shift register corresponds to contents of the delay components z^{-1} of the structure in

Figure 5. For $k=0$ register D contains $x(-n)$, otherwise it is connected to the output of the shift register. For $k=0$ and $k=1$ register C is cleared, otherwise it is connected to the output of ALU. The length of the shift register is equal to the length of the input sequence. Alternatively, random-access memory can be employed to provide date storage.

4. Application of SW in Testing

Spectral warping technique originally developed for speech correction can find new interesting and efficient applications in the emerging area of *Analog and Mixed-Signal Testing* [7]. For example, the network can be used when testing is realized by means of traditional external test equipment. Such an application is of rather straightforward manner. It can offer reasonable benefits in mass production testing where high test throughput is vitally important. The use of frequency spectral warping pre-processing to obtain unequal spectral resolution could reduce total computing times and thus time to perform testing. This could be especially important in testing of high-speed high-frequency circuits and systems.

The other area where application of digital spectral warping could lead even to more substantial benefits is built-in self-test (BIST). In short, the approach implies that control, test signal generation, response measurement and analysis are performed by units incorporated into DUT itself. They provide on-site testing of DUT internal circuitry, and deliver the results to its external terminals. One of the current trends in mixed-signal BIST is to use digital tools and methods to test both digital and analog components [1,5-8]. This approach is often called *DSP-Based Testing*. Basic structure of DSP-based testing for mixed-signal DUT is presented in Figure 7 [8].

It can be seen that both signal generation as well as output measurement and analysis are realized by means of the digital circuitry. Among the test signals that can be used in the scheme are digitized sinusoid, digitized multi-tone, pseudo-random signals, and some others. The following algorithms can be used for response analysis: Discrete Fourier Transform computing by means of Fast Fourier Transform (FFT), IEEE 1057 sine-wave fitting, cross- and auto-correlation, etc. [5, 8]. Such an organization is claimed to be an effective way around many otherwise unavoidable limits of analog instrumentation (cross-talk, non-linearity, noise, drift, aging, improper calibration, long filter setting time, thermal effects and so on) while providing benefits inherent to the use of digital components (accuracy, stability, single set up for multiple types of tests, results repeatability, etc.). Besides, in DSP-based testing all test access can be obtained through the same digital I/O ports, while DUT can be tested for many parameters in one run thus increasing throughput. The speed of DSP testing depends heavily on the speed of the processor. Such dependence can be a serious weak point of the scheme (due to the limitations on the computing resources inherent to built-in test resources). The use of spectral warping can significantly ease the problem, thus making DSP-based testing more efficient.

Spectral warping can naturally be employed for test response analysis when FFT processing is used and when non-uniform frequency resolution is required. As it was mentioned earlier its application allows to decrease the data array size for FFT, and thereby to increase performance of its computation. Besides, combined with *Pseudo-Random Test Pattern Generation (PRTPG)*, Inverse DFT, or digital multi-tone generation [5, 8] the warping network can be used to realize generation of test signals with pre-specified non-uniform spectral characteristics, for example having some "focused" or "stretched" spectrum, that could be of particular importance in telecom or AV circuits and systems testing. The ability to use the same hardware for both generation and analysis is an additional substantial advantage of such an approach.

In both the cases (test stimuli generation and response analysis) the presented in Section 2 amplitude-frequency and phase-frequency characteristics of the network have to be taken into account in order to secure obtaining correct results of testing.

Let, for example, the SW is used in DUT transfer function testing with non-linear frequency resolution using FFT technique (Fig. 8). The signal applied to the DUT from the signal source having known frequency spectrum $S_x(f)$ and power σ_0^2. The signal output from the DUT is converted from analog to digital form and transformed by FFT into frequency spectra $S_y(f)$. In order to limit the measurement errors caused by the DUT non-linearity and by internal noise within the DUT itself the ensemble averaging can be used, and the transfer function is found as

$$H(f) = \frac{< S_y(f) S^*{}_x(f) >}{\sigma_0^2} \tag{7}$$

where $<>$ - ensemble averaging, $S^*_x(f)$ is the complex conjugate of $S_x(f)$.

In this application SW is used for both generation of unequally spaced frequencies (with some value of warping parameter a), and for output spectral analysis (with the warping parameter equal to $-a$ as the frequencies have to be placed back into the uniform mode). Thus the total distortion introduced by the double warping is

$$\left|H_M(\varpi)\right| = \frac{(1-a^2)^2}{(1+a^2)^2 - 4a^2\cos^2 wT} \tag{8}$$

To get rid of the amplitude distortion, special correction by means of a digital filter having amplitude-frequency characteristics inverse to $H_M(\omega)$ can be employed. The structure of such modified spectral warping network is presented in Fig. 8. The non-recursive digital filtering algorithm is employed to correct the amplitude characteristic distortions. The filter coefficients can be calculated in advance for the given value of a, or they can be found on the spot by using on-chip FFT processor (by performing inverse Fourier transform). At the first stage of the system operation input signal samples are entered into the shift register and correction digital filtering is realized by employing *ALU*, multiplexers, shift register and register *D*. After that spectral warping algorithm is performed in the same way as it was described in [2].

In the proposed structure both the network and the correcting filter use the same shift register. Besides since PRTPG is usually realized as a *Linear Feedback Shift Register* it also can be implemented as in integrated part of the design on the basis of the same register. This opens up possibilities for substantial minimization in terms of the volume of hardware used. In addition significant hardware savings can be achieved by using memory elements of the *Boundary Scan* (if it is presented in the device-under-test) to implement the SW network/filter/generator.

5. Conclusion

In this paper a special algorithm for discrete representation of discrete-time signal is discussed, corresponding to the representation of one digital sequence by another digital sequence. The algorithm (called Spectral Warping) based on the concept of mapping the z-transform by substitution of variables. It allows maintaining the *shape* of spectrum for a discrete-time signal, but transform the frequency axis in a non-linear manner.

This feature has potential application in analog and mixed-signal electronic testing contexts. In particular, the spectral warping can be employed for the test signal generation and test response analysis where non-uniform frequency resolution is required (e.g., testing with logarithmic resolution – octave analysis). It allows to decrease the array size for DFT or other frequency analysis techniques used, and thereby to increase test throughput as measured, for example, by the speed of test execution. However the spectral warping transform is characterized by substantial frequency-dependent amplification. It has to be taken into account or compensated when using the algorithm in measurement practice. One of the possible ways of such compensation is to use digital pre-filtering with transfer function that is inverse to that of the SW network employed.

Our paper presents just an initial discussion of how the application of SW could benefit to analog and mixed-signal testing. A number of theoretical and practical problems have to be solved before the application of spectral warping will actually come into practice of engineering testing. The research on the area is already under way, and we hope to publish a summary on the results in due course.

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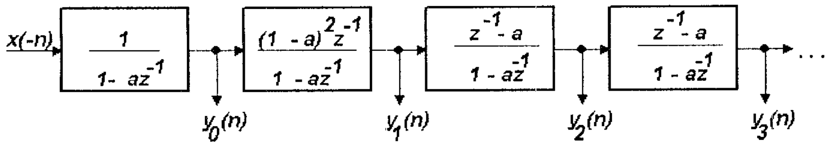

Figure 1. Spectral Warping Network

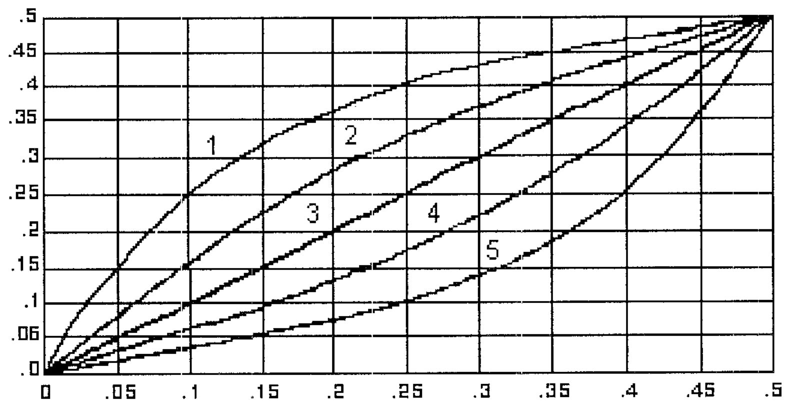

Figure 2. Frequency warping transformation for *a*=0.5, 0.25, 0.0, -0.25, and –0.5 (normalized by the sampling frequency)

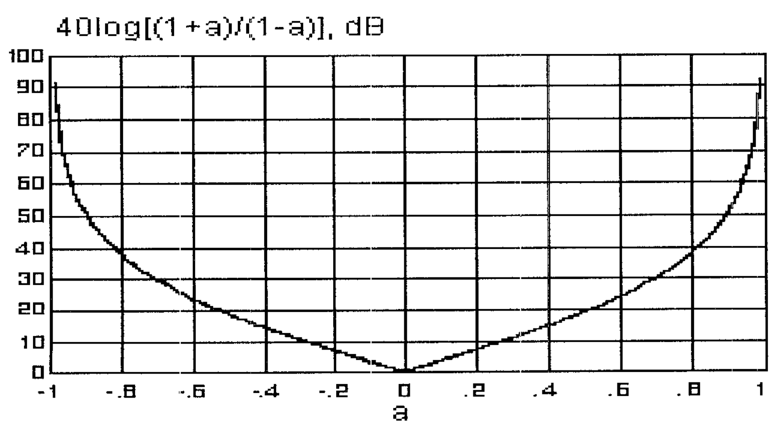

Figure 3. Attenuation at the half sampling frequency (relative to unity *dc* gain)

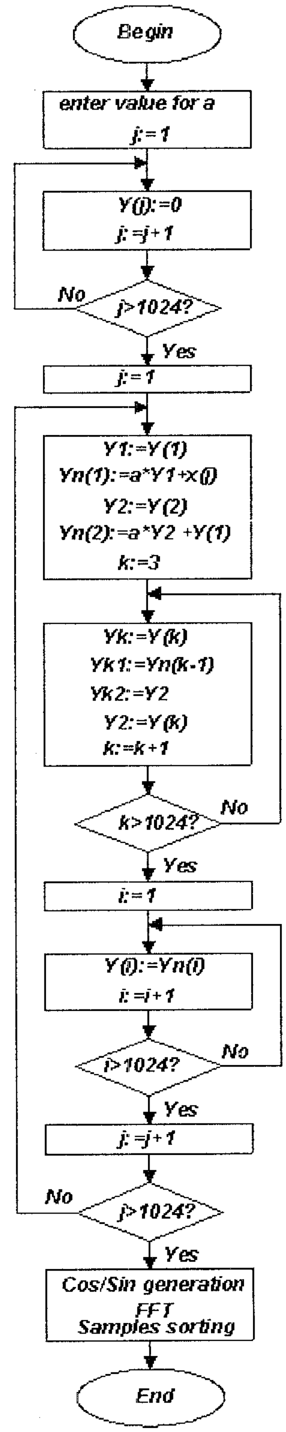

Figure 4. Spectral warping algorithm for 1024 samples

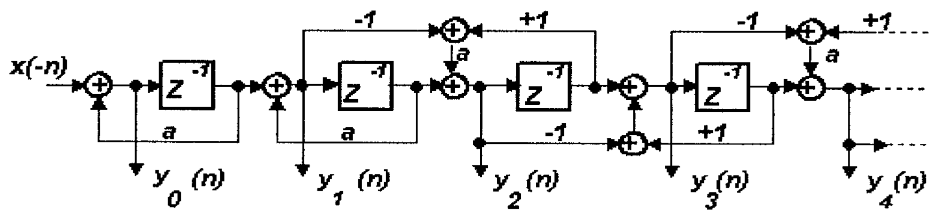

Figure 5. Spectral warping network architecture

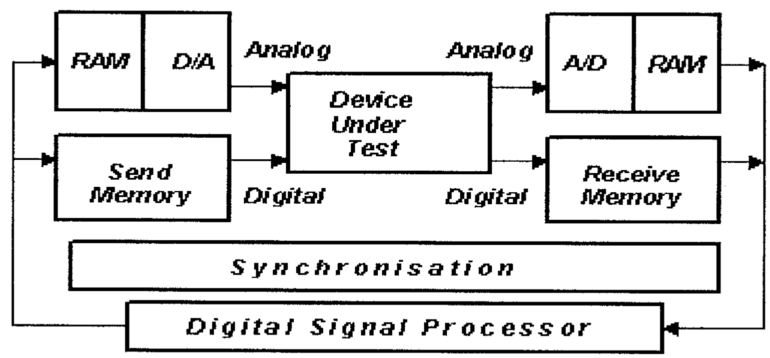

Figure 6. Block diagram of hardware implementation of the network

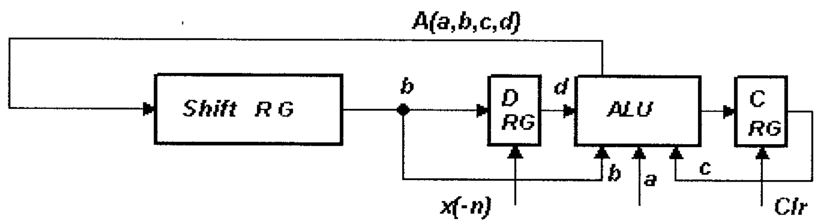

Figure 7. DSP-based testing of mixed-signal circuit

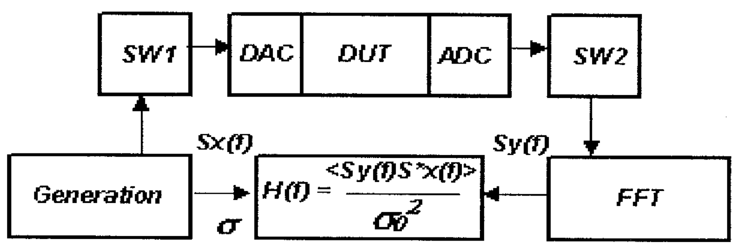

Fig. 8. Transfer function testing

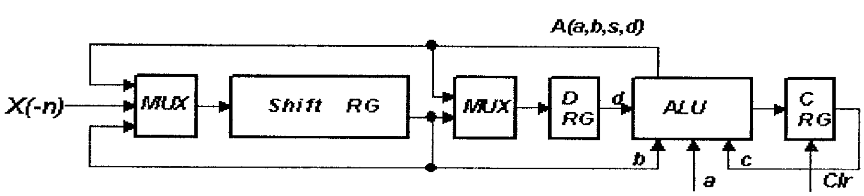

Fig. 9. SW network with amplitude-frequency correction